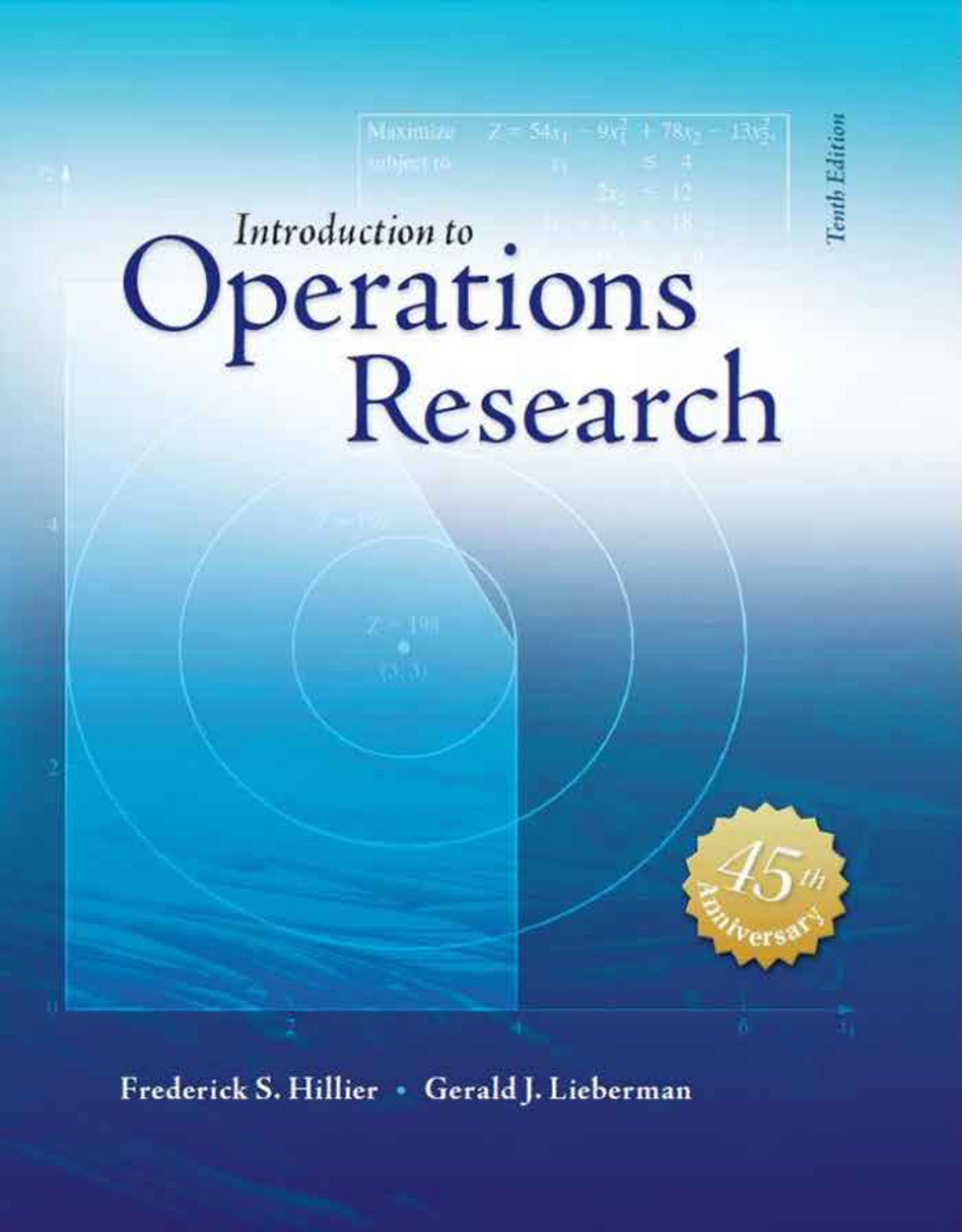
Introduction to
Operations Research
Tenth Edition
45th Anniversary
Frederick S. Hillier • Gerald J. Lieberman

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INTRODUCTION TO OPERATIONS RESEARCH

INTRODUCTION TO OPERATIONS RESEARCH

Tenth Edition

FREDERICK S. HILLIER
Stanford University

GERALD J. LIEBERMAN
Late of Stanford University

Mc Graw Hill Education

INTRODUCTION TO OPERATIONS RESEARCH, TENTH EDITION

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ABOUT THE AUTHORS

Frederick S. Hillier was born and raised in Aberdeen, Washington, where he was an award winner in statewide high school contests in essay writing, mathematics, debate, and music. As an undergraduate at Stanford University, he ranked first in his engineering class of over 300 students. He also won the McKinsey Prize for technical writing, won the Outstanding Sophomore Debater award, played in the Stanford Woodwind Quintet and Stanford Symphony Orchestra, and won the Hamilton Award for combining excellence in engineering with notable achievements in the humanities and social sciences. Upon his graduation with a BS degree in industrial engineering, he was awarded three national fellowships (National Science Foundation, Tau Beta Pi, and Danforth) for graduate study at Stanford with specialization in operations research. During his three years of graduate study, he took numerous additional courses in mathematics, statistics, and economics beyond what was required for his MS and PhD degrees while also teaching two courses (including "Introduction to Operations Research"). Upon receiving his PhD degree, he joined the faculty of Stanford University and began work on the 1st edition of this textbook two years later. He subsequently earned tenure at the age of 28 and the rank of full professor at 32. He also received visiting appointments at Cornell University, Carnegie-Mellon University, the Technical University of Denmark, the University of Canterbury (New Zealand), and the University of Cambridge (England). After 35 years on the Stanford faculty, he took early retirement from his faculty responsibilities in order to focus full time on textbook writing, and now is Professor Emeritus of Operations Research at Stanford.

Dr. Hillier's research has extended into a variety of areas, including integer programming, queueing theory and its application, statistical quality control, and the application of operations research to the design of production systems and to capital budgeting. He has published widely, and his seminal papers have been selected for republication in books of selected readings at least 10 times. He was the first-prize winner of a research contest on "Capital Budgeting of Interrelated Projects" sponsored by The Institute of Management Sciences (TIMS) and the U.S. Office of Naval Research. He and Dr. Lieberman also received the honorable mention award for the 1995 Lanchester Prize (best English-language publication of any kind in the field of operations research), which was awarded by the Institute of Operations Research and the Management Sciences (INFORMS) for the 6th edition of this book. In addition, he was the recipient of the prestigious 2004 INFORMS Expository Writing Award for the 8th edition of this book.

Dr. Hillier has held many leadership positions with the professional societies in his field. For example, he has served as treasurer of the Operations Research Society of America (ORSA), vice president for meetings of TIMS, co-general chairman of the 1989 TIMS International Meeting in Osaka, Japan, chair of the TIMS Publications Committee, chair of the ORSA Search Committee for Editor of *Operations Research,* chair of the ORSA Resources Planning Committee, chair of the ORSA/TIMS Combined Meetings Committee, and chair of the John von Neumann Theory Prize Selection Committee for INFORMS. He also is a Fellow of INFORMS. In addition, he recently completed a 20-year tenure as the series editor for Springer's International Series in Operations Research and Management Science, a particularly prominent book series with over 200 published books that he founded in 1993.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *The Evaluation of Risky Interrelated Investments* (North-Holland, 1969), *Queueing Tables and Graphs* (Elsevier North-Holland, 1981, co-authored by O. S. Yu, with D. M. Avis, L. D. Fossett, F. D. Lo, and M. I. Reiman), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (5th ed., McGraw-Hill/Irwin, 2014, co-authored by his son Mark Hillier).

The late **Gerald J. Lieberman** sadly passed away in 1999. He had been Professor Emeritus of Operations Research and Statistics at Stanford University, where he was the founding chair of the Department of Operations Research. He was both an engineer (having received an undergraduate degree in mechanical engineering from Cooper Union) and an operations research statistician (with an AM from Columbia University in mathematical statistics, and a PhD from Stanford University in statistics).

Dr. Lieberman was one of Stanford's most eminent leaders in recent decades. After chairing the Department of Operations Research, he served as associate dean of the School of Humanities and Sciences, vice provost and dean of research, vice provost and dean of graduate studies, chair of the faculty senate, member of the University Advisory Board, and chair of the Centennial Celebration Committee. He also served as provost or acting provost under three different Stanford presidents.

Throughout these years of university leadership, he also remained active professionally. His research was in the stochastic areas of operations research, often at the interface of applied probability and statistics. He published extensively in the areas of reliability and quality control, and in the modeling of complex systems, including their optimal design, when resources are limited.

Highly respected as a senior statesman of the field of operations research, Dr. Lieberman served in numerous leadership roles, including as the elected president of The Institute of Management Sciences. His professional honors included being elected to the National Academy of Engineering, receiving the Shewhart Medal of the American Society for Quality Control, receiving the Cuthbertson Award for exceptional service to Stanford University, and serving as a fellow at the Center for Advanced Study in the Behavioral Sciences. In addition, the Institute of Operations Research and the Management Sciences (INFORMS) awarded him and Dr. Hillier the honorable mention award for the 1995 Lanchester Prize for the 6th edition of this book. In 1996, INFORMS also awarded him the prestigious Kimball Medal for his exceptional contributions to the field of operations research and management science.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *Handbook of Industrial Statistics* (Prentice-Hall, 1955, co-authored by A. H. Bowker), *Tables of the Non-Central t-Distribution* (Stanford University Press, 1957, co-authored by G. J. Resnikoff), *Tables of the Hypergeometric Probability Distribution* (Stanford University Press, 1961, co-authored by D. Owen), *Engineering Statistics*, (2nd ed., Prentice-Hall, 1972, co-authored by A. H. Bowker), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (McGraw-Hill/Irwin, 2000, co-authored by F. S. Hillier and M. S. Hillier).

ABOUT THE CASE WRITERS

Karl Schmedders is professor of quantitative business administration at the University of Zurich in Switzerland and a visiting associate professor at the Kellogg Graduate School of Management (Northwestern University). His research interests include management science, financial economics, and computational economics and finance. in 2003, a paper by Dr. Schmedders received a nomination for the Smith-Breeden Prize for the best paper in *Journal of Finance*. He received his doctorate in operations research from Stanford University, where he taught both undergraduate and graduate classes in operations research, including a case studies course in operations research. He received several teaching awards at Stanford, including the university's prestigious Walter J. Gores Teaching Award. After post-doctoral research at the Hoover Institution, a think tank on the Stanford campus, he became assistant professor of managerial economics and decision sciences at the Kellogg School. He was promoted to associate professor in 2001 and received tenure in 2005. In 2008, he joined the University of Zurich, where he currently teaches courses in management science, spreadsheet modeling, and computational economics and finance. At Kellogg he received several teaching awards, including the L. G. Lavengood Professor of the Year Award. More recently he won the best professor award of the Kellogg School's European EMBA program (2008, 2009, and 2011) and its Miami EMBA program (2011).

Molly Stephens is a partner in the Los Angeles office of Quinn, Emanuel, Urquhart & Sullivan, LLP. She graduated from Stanford University with a BS degree in industrial engineering and an MS degree in operations research. Ms. Stephens taught public speaking in Stanford's School of Engineering and served as a teaching assistant for a case studies course in operations research. As a teaching assistant, she analyzed operations research problems encountered in the real world and the transformation of these problems into classroom case studies. Her research was rewarded when she won an undergraduate research grant from Stanford to continue her work and was invited to speak at an INFORMS conference to present her conclusions regarding successful classroom case studies. Following graduation, Ms. Stephens worked at Andersen Consulting as a systems integrator, experiencing real cases from the inside, before resuming her graduate studies to earn a JD degree (with honors) from the University of Texas Law School at Austin. She is a partner in the largest law firm in the United States devoted solely to business litigation, where her practice focuses on complex financial and securities litigation.

DEDICATION

To the memory of our parents

and

To the memory of my beloved mentor,
Gerald J. Lieberman, who was one of the true
giants of our field

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PREFACE

When Jerry Lieberman and I started working on the first edition of this book 50 years ago, our goal was to develop a pathbreaking textbook that would help establish the future direction of education in what was then the emerging field of operations research. Following publication, it was unclear how well this particular goal was met, but what did become clear was that the demand for the book was far larger than either of us had anticipated. Neither of us could have imagined that this extensive worldwide demand would continue at such a high level for such an extended period of time.

The enthusiastic response to our first nine editions has been most gratifying. It was a particular pleasure to have the field's leading professional society, the international Institute for Operations Research and the Management Sciences (INFORMS), award the 6th edition honorable mention for the 1995 INFORMS Lanchester Prize (the prize awarded for the year's most outstanding English-language publication of any kind in the field of operations research).

Then, just after the publication of the eighth edition, it was especially gratifying to be the recipient of the prestigious 2004 INFORMS Expository Writing Award for this book, including receiving the following citation:

> Over 37 years, successive editions of this book have introduced more than one-half million students to the field and have attracted many people to enter the field for academic activity and professional practice. Many leaders in the field and many current instructors first learned about the field via an edition of this book. The extensive use of international student editions and translations into 15 other languages has contributed to spreading the field around the world. The book remains preeminent even after 37 years. Although the eighth edition just appeared, the seventh edition had 46 percent of the market for books of its kind, and it ranked second in international sales among all McGraw-Hill publications in engineering.
>
> Two features account for this success. First, the editions have been outstanding from students' points of view due to excellent motivation, clear and intuitive explanations, good examples of professional practice, excellent organization of material, very useful supporting software, and appropriate but not excessive mathematics. Second, the editions have been attractive from instructors' points of view because they repeatedly infuse state-of-the-art material with remarkable lucidity and plain language. For example, a wonderful chapter on metaheuristics was created for the eighth edition.

When we began work on the book 50 years ago, Jerry already was a prominent member of the field, a successful textbook writer, and the chairman of a renowned operations research program at Stanford University. I was a very young assistant professor just starting my career. It was a wonderful opportunity for me to work with and to learn from the master. I will be forever indebted to Jerry for giving me this opportunity.

Now, sadly, Jerry is no longer with us. During the progressive illness that led to his death 14 years ago, I resolved that I would pick up the torch and devote myself to subsequent editions of this book, maintaining a standard that would fully honor Jerry. Therefore, I took early retirement from my faculty responsibilities at Stanford in order to work full time on textbook writing for the foreseeable future. This has enabled me to spend far more than the usual amount of time in preparing each new edition. It also has enabled me to closely monitor new trends and developments in the field in order to bring this edition completely up to date. This monitoring has led to the choice of the major additions to the new edition outlined next.

WHAT'S NEW IN THIS EDITION

- **Analytic Solver Platform for Education.** This edition continues to provide the option of using Excel and its Solver (a product of Frontline Systems, Inc.) to formulate and solve some operations research (OR) models. Frontline Systems also has developed some advanced Excel-based software packages. One recently released package, Analytic Solver Platform, is particularly exciting because of its tremendous versatility. It provides strong capability for dealing with the types of OR models considered in most of the chapters considered in this book, including linear programming, integer programming, nonlinear programming, decision analysis, simulation, and forecasting. Rather than requiring the use of a collection of Excel add-ins to deal with all of these areas (as in the preceding edition), Analytic Solver Platform provides an all-in-one package for formulating and solving many OR models in spreadsheets. We are delighted to have integrated the student version of this package, Analytic Solver Platform for Education (ASPE), into this new edition. A special arrangement has been made with Frontline Systems to provide students with a free 140-day license for ASPE.

  At the same time, we have integrated ASPE in such a way that it can readily be skipped over without loss of continuity for those who do not wish to use spreadsheets. A number of other attractive software options continue to be provided in this edition (as described later). In addition, a relatively brief introduction to spreadsheet modeling can also be obtained by only using Excel's standard Solver. However, we believe that many instructors and students will welcome the great power and versatility of ASPE.
- **A New Section on Robust Optimization**. OR models typically are formulated to help select some future course of action, so the values of the model parameters need to be based on a prediction of future conditions. This sometimes results in having a significant amount of uncertainty about what the parameter values actually will turn out to be when the optimal solution from the model is implemented. For problems where there is no latitude for violating the constraints even a little bit, a relatively new technique called *robust optimization* provides a way of obtaining a solution that is virtually guaranteed to be feasible and nearly optimal regardless of reasonable deviations of the parameter values from their estimated values. The new Section 7.4 introduces the robust optimization approach when dealing with linear programming problems.
- **A New Section on Chance Constraints.** The new Section 7.5 continues the discussion in Section 7.4 by turning to the case where there is some latitude for violating some constraints a little bit without very serious complications. This leads to the option of using *chance constraints*, where each chance constraint modifies an original constraint by only requiring that there be some very high probability that the original constraint will be satisfied. When the original problem is a linear programming problem, each of these chance constraints can be converted into a deterministic equivalent that still is a linear programming constraint. Section 7.5 describes how this important idea is implemented.
- **A New Section on Stochastic Programming with Recourse**. Stochastic programming provides still another way of reformulating a linear programming model (or another type of model) where there is some uncertainty about what the values of the parameters will turn out to be. This approach is particularly valuable for those problems where the decisions will be made in two (or more) stages, so the decisions in stage 2 can help compensate for any stage 1 decisions that do not turn out as well as hoped because of errors in estimating some parameter values. The new Section 7.6 describes *stochastic programming with recourse* for dealing with such problems.
- **A New Chapter on Linear Programming under Uncertainty That Includes These New Sections.** One of the key assumptions of linear programming (as for many other OR models) is the *certainty assumption*, which says that the value assigned to each parameter

of a linear programming model is assumed to be a *known constant*. This is a convenient assumption, but it seldom is satisfied precisely. One of the most important concepts to get across in an introductory OR course is that (1) although it usually is necessary to make some simplifying assumptions when formulating a model of a problem, (2) it then is very important after solving the model to explore the impact of these simplifying assumptions. This concept can be most readily conveyed in the context of linear programming because of all the methodology that now has been developed for dealing with linear programming under uncertainty. One key technique of this type is sensitivity analysis, but some other relatively elementary techniques now have also been well developed, including particularly the ones presented in the three new sections described above. Therefore, the old Chapter 6 (*Duality Theory and Sensitivity Analysis*) now has been divided into two new chapters—Chapter 6 (*Duality Theory*) and Chapter 7 (*Linear Programming under Uncertainty*). The new Chapter 7 includes the three sections on sensitivity analysis in the old Chapter 6 but also adds the three new sections described above.

- **A New Section on the Rise of Analytics Together with Operations Research.** A particularly dramatic development in the field of operations research over the last several years has been the great buzz throughout the business world about something called *analytics* (or business analytics) and the importance of incorporating analytics into managerial decision making. As it turns out, the discipline of analytics is closely related to the discipline of operations research, although there are some differences in emphases. OR can be thought of as focusing mainly on advanced analytics whereas analytics professionals might get more involved with less advanced aspects of the study. Some fads come and go, but this appears to be a permanent shift in the direction of OR in the coming years. In fact, we could even find *analytics* eventually replacing *operations research* as the common name for this integrated discipline. Because of this close and growing tie between the two disciplines, it has become important to describe this relationship and to put it into perspective in an introductory OR course. This has been done in the new Section 1.3.
- **Many New or Revised Problems.** A significant number of new problems have been added to support the new topics and application vignettes. In addition, many of the problems from the ninth edition have been revised. Therefore, an instructor who does not wish to assign problems that were assigned in previous classes has a substantial number from which to choose.
- **A Reorganization to Reduce the Size of the Book.** An unfortunate trend with early editions of this book was that each new edition was significantly larger than the previous one. This continued until the seventh edition had become considerably larger than is desirable for an introductory survey textbook. Therefore, I worked hard to substantially reduce the size of the eighth edition and and then further reduced the size of the ninth edition slightly. I also adopted the goal of avoiding any growth in subsequent editions. Indeed, this edition is 35 pages shorter than the ninth edition. This was accomplished through a variety of means. One was being careful not to add too much new material. Another was deleting certain low-priority material, including the presentation of parametric linear programming in conjunction with sensitivity analysis (it already is covered later in Section 8.2) and a complicated dynamic programming example (the Wyndor problem with three state variables) that can be solved much more easily in other ways. Finally, and most importantly, 50 pages were saved by shifting two little-used items (the chapter on Markov chains and the last two major sections on Markov decision processes) to the supplements on the book's website. Markov chains are a central topic of probability theory and stochastic processes that have been borrowed as a tool of operations research, so this chapter better fits as a reference in the supplements.
- **Updating to Reflect the Current State of the Art.** A special effort has been made to keep the book completely up to date. This included adding relatively new developments (the four new sections mentioned above) that now warrant consideration in an

introductory survey course, as well as making sure that all the material in the ninth edition has been brought up to date. It also included carefully updating both the application vignettes and selected references for each chapter.

OTHER SPECIAL FEATURES OF THIS BOOK

- **An Emphasis on Real Applications.** The field of operations research is continuing to have a dramatic impact on the success of numerous companies and organizations around the world. Therefore, one of the goals of this book is to tell this story clearly and thereby excite students about the great relevance of the material they are studying. This goal is pursued in four ways. One is the inclusion of many application vignettes scattered throughout the book that describe in a few paragraphs how an actual application of operations research had a powerful impact on a company or organization by using techniques like those studied in that portion of the book. For each application vignette, a problem also is included in the problems section of that chapter that requires the student to read the full article describing the application and then answer some questions. Second, real applications also are briefly described (especially in Chapters 2 and 12) as part of the presentation of some OR technique to illustrate its use. Third, many cases patterned after real applications are included at the end of chapters and on the book's website. Fourth, many selected references of award winning OR applications are given at the end of some of the chapters. Once again, problems are included at the end of these chapters that require reading one or more of the articles describing these applications. The next bullet point describes how students have immediate access to these articles.
- **Links to Many Articles Describing Dramatic OR Applications.** We are excited about a partnership with The Institute for Operations Research and the Management Sciences (INFORMS), our field's preeminent professional society, to provide a link on this book's website to approximately 100 articles describing award winning OR applications, including the ones described in all of the application vignettes. (Information about INFORMS journals, meetings, job bank, scholarships, awards, and teaching materials is at www.informs.org.) These articles and the corresponding end-of-chapter problems provide instructors with the option of having their students delve into real applications that dramatically demonstrate the relevance of the material being covered in the lectures. It would even be possible to devote significant course time to discussing real applications.
- **A Wealth of Supplementary Chapters and Sections on the Website.** In addition to the approximately 1,000 pages in this book, another several hundred pages of supplementary material also are provided on this book's website (as outlined in the table of contents). This includes nine complete chapters and a considerable number of supplements to chapters in the book, as well as a substantial number of additional cases. All of the supplementary chapters include problems and selected references. Most of the supplements to chapters also have problems. Today, when students think nothing of accessing material electronically, instructors should feel free to include some of this supplementary material in their courses.
- **Many Additional Examples Are Available.** An especially important learning aid on the book's website is a set of Solved Examples for almost every chapter in the book. We believe that most students will find the examples in the book fully adequate but that others will feel the need to go through additional examples. These solved examples on the website will provide the latter category of students the needed help, but without interrupting the flow of the material in the book on those many occasions when most students don't need to see an additional example. Many students also might find these additional examples helpful when preparing for an examination. We recommend to instructors that they point out this important learning aid to their students.

- **Great Flexibility for What to Emphasize.** We have found that there is great variability in what instructors want to emphasize in an introductory OR survey course. They might want to emphasize the mathematics and algorithms of operations research. Others will emphasize model formulation with little concern for the details of the algorithms needed to solve these models. Others want an even more applied course, with emphasis on applications and the role of OR in managerial decision making. Some instructors will focus on the deterministic models of OR, while others will emphasize stochastic models. There also are great differences in the kind of software (if any) that instructors want their students to use. All of this helps to explain why the book is a relatively large one. We believe that we have provided enough material to meet the needs of all of these kinds of instructors. Furthermore, the book is organized in such a way that it is relatively easy to pick and choose the desired material without loss of continuity. It even is possible to provide great flexibility on the kind of software (if any) that instructors want their students to use, as described below in the section on software options.
- **A Customizable Version of the Text Also is Available.** Because the text provides great flexibility for what to emphasize, an instructor can easily pick and choose just certain portions of the book to cover. Rather than covering nearly all of the 1,000 pages in the book, perhaps you wish to use only a much smaller portion of the text. Fortunately, McGraw-Hill provides an option for using a considerably smaller and less expensive version of the book that is customized to meet your needs. With McGraw-Hill Create™, you can include only the chapters you want to cover. You also can easily rearrange chapters, combine material from other content sources, and quickly upload content you have written, like your course syllabus or teaching notes. If desired, you can use Create to search for useful supplementary material in various other leading McGraw-Hill textbooks. For example, if you wish to emphasize spreadsheet modeling and applications, we would recommend including some chapters from the Hillier-Hillier textbook, *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets*. Arrange your book to fit your teaching style. Create even allows you to personalize your book's appearance by selecting the cover and adding your name, school, and course information. Order a Create book and you'll receive a complimentary print review copy in 3–5 business days or a complimentary electronic review copy (eComp) via e-mail in minutes. You can go to www.mcgrawhillcreate.com and register to experience how McGraw-Hill Create empowers you to teach your students your way.

A WEALTH OF SOFTWARE OPTIONS

A wealth of software options is provided on the book's website www.mhhe.com/hillier as outlined below:

- Excel spreadsheets: state-of-the-art spreadsheet formulations in Excel files for all relevant examples throughout the book. The standard Excel Solver can solve most of these examples.
- As described earlier, the powerful Analytic Solver Platform for Education (ASPE) to formulate and solve a wide variety of OR models in an Excel environment.
- A number of Excel templates for solving basic models.
- Student versions of LINDO (a traditional optimizer) and LINGO (a popular algebraic modeling language), along with formulations and solutions for all relevant examples throughout the book.
- Student versions of MPL (a leading algebraic modeling language) along with an MPL Tutorial and MPL formulations and solutions for all relevant examples throughout the book.
- Student versions of several elite MPL solvers for linear programming, integer programming, convex programming, global optimization, etc.
- Queueing Simulator (for the simulation of queueing systems).

- OR Tutor for illustrating various algorithms in action.
- Interactive Operations Research (IOR) Tutorial for efficiently learning and executing algorithms interactively, implemented in Java 2 in order to be platform independent.

Numerous students have found OR Tutor and IOR Tutorial very helpful for learning algorithms of operations research. When moving to the next stage of solving OR models automatically, surveys have found instructors almost equally split in preferring one of the following options for their students' use: (1) Excel spreadsheets, including Excel's Solver (and now ASPE), (2) convenient traditional software (LINDO and LINGO), and (3) state-of-the-art OR software (MPL and its elite solvers). For this edition, therefore, I have retained the philosophy of the last few editions of providing enough introduction in the book to enable the basic use of any of the three options without distracting those using another, while also providing ample supporting material for each option on the book's website.

Because of the power and versatility of ASPE, we no longer include a number of Excel-based software packages (Crystal Ball, Premium Solver for Education, TreePlan, SensIt, RiskSim, and Solver Table) that were bundled with recent editions. ASPE alone matches or exceeds the capabilities of all these previous packages.

Additional Online Resources

- A *glossary* for every book chapter.
- *Data files* for various cases to enable students to focus on analysis rather than inputting large data sets.
- A *test bank* featuring moderately difficult questions that require students to show their work is being provided to instructors. Many of the questions in this test bank have previously been used successfully as test questions by the authors. The test bank for this new edition has been greatly expanded from the one for the 9th edition, so many new test questions now are available to instructors.
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THE USE OF THE BOOK

The overall thrust of all the revision efforts has been to build upon the strengths of previous editions to more fully meet the needs of today's students. These revisions make the book even more suitable for use in a modern course that reflects contemporary practice in the field. The use of software is integral to the practice of operations research, so the wealth of software options accompanying the book provides great flexibility to the instructor in choosing the preferred types of software for student use. All the educational resources accompanying the book further enhance the learning experience. Therefore, the book and its website should fit a course where the instructor wants the students to have a single self-contained textbook that complements and supports what happens in the classroom.

The McGraw-Hill editorial team and I think that the net effect of the revision has been to make this edition even more of a "student's book"—clear, interesting, and well-organized with lots of helpful examples and illustrations, good motivation and perspective, easy-to-find important material, and enjoyable homework, without too much notation, terminology, and dense mathematics. We believe and trust that the numerous instructors who have used previous editions will agree that this is the best edition yet.

The prerequisites for a course using this book can be relatively modest. As with previous editions, the mathematics has been kept at a relatively elementary level. Most of Chaps. 1 to 15 (introduction, linear programming, and mathematical programming) require no mathematics beyond high school algebra. Calculus is used only in Chap. 13 (Nonlinear Programming) and in one example in Chap. 11 (Dynamic Programming). Matrix notation is used in Chap. 5 (The Theory of the Simplex Method), Chap. 6 (Duality Theory), Chap. 7 (Linear Programming under Uncertainty), Sec. 8.4 (An Interior-Point Algorithm), and Chap. 13, but the only background needed for this is presented in Appendix 4. For Chaps. 16 to 20 (probabilistic models), a previous introduction to probability theory is assumed, and calculus is used in a few places. In general terms, the mathematical maturity that a student achieves through taking an elementary calculus course is useful throughout Chaps. 16 to 20 and for the more advanced material in the preceding chapters.

The content of the book is aimed largely at the upper-division undergraduate level (including well-prepared sophomores) and at first-year (master's level) graduate students. Because of the book's great flexibility, there are many ways to package the material into a course. Chapters 1 and 2 give an introduction to the subject of operations research. Chapters 3 to 15 (on linear programming and mathematical programming) may essentially be covered independently of Chaps. 16 to 20 (on probabilistic models), and vice-versa. Furthermore, the individual chapters among Chaps. 3 to 15 are almost independent, except that they all use basic material presented in Chap. 3 and perhaps in Chap. 4. Chapters 6 and 7 and Sec. 8.2 also draw upon Chap. 5. Sections 8.1 and 8.2 use parts of Chaps. 6 and 7. Section 10.6 assumes an acquaintance with the problem formulations in Secs. 9.1 and 9.3, while prior exposure to Secs. 8.3 and 9.2 is helpful (but not essential) in Sec. 10.7. Within Chaps. 16 to 20, there is considerable flexibility of coverage, although some integration of the material is available.

An elementary survey course covering linear programming, mathematical programming, and some probabilistic models can be presented in a quarter (40 hours) or semester by selectively drawing from material throughout the book. For example, a good survey of the field can be obtained from Chaps. 1, 2, 3, 4, 16, 17, 18, and 20, along with parts of Chaps. 10 to 14. A more extensive elementary survey course can be completed in two quarters (60 to 80 hours) by excluding just a few chapters, for example, Chaps. 8, 15, and 19. Chapters 1 to 9 (and perhaps part of Chap. 10) form an excellent basis for a (one-quarter) course in linear programming. The material in Chaps. 10 to 15 covers topics for another (one-quarter) course in other deterministic models. Finally, the material in Chaps. 16 to 20 covers the probabilistic (stochastic) models of operations research suitable for presentation in a (one-quarter) course. In fact, these latter three courses (the material in the entire text) can be viewed as a basic one-year sequence in the techniques of operations research, forming the core of a master's degree program. Each course outlined has been presented at either the undergraduate or graduate level at Stanford University, and this text has been used in basically the manner suggested.

The book's website will provide updates about the book, including an errata. To access this site, visit www.mhhe.com/hillier.

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This edition was very much of a team effort. Our case writers, Karl Schmedders and Molly Stephens (both graduates of our department), wrote 24 elaborate cases for the 7th edition, and all of these cases continue to accompany this new edition. One of our department's former PhD students, Michael O'Sullivan, developed OR Tutor for the 7th edition (and continued here), based on part of the software that my son Mark Hillier had developed

for the 5th and 6th editions. Mark (who was born the same year as the first edition, earned his PhD at Stanford, and now is a tenured Associate Professor of Quantitative Methods at the University of Washington) provided both the spreadsheets and the Excel files (including many Excel templates) once again for this edition, as well as the Queueing Simulator. He also gave important help on the textual material involving ASPE and contributed greatly to Chaps. 21 and 28 on the book's website. In addition, he updated the 10th edition version of the solutions manual. Earlier editions of this solutions manual were prepared in an exemplary manner by a long sequence of PhD students from our department, including Che-Lin Su for the 8th edition and Pelin Canbolat for the 9th edition. Che-Lin and Pelin did outstanding work that nicely paved the way for Mark's work on the solutions manual. Last, but definitely not least, my dear wife, Ann Hillier (another Stanford graduate with a minor in operations research), provided me with important help on an almost daily basis. All the individuals named above were vital members of the team.

I also owe a great debt of gratitude to four individuals and their companies for providing the special software and related information for the book. Another Stanford PhD graduate, William Sun (CEO of the software company Accelet Corporation), and his team did a brilliant job of starting with much of Mark Hillier's earlier software and implementing it anew in Java 2 as IOR Tutorial for the 7th edition, as well as further enhancing IOR Tutorial for the subsequent editions. Linus Schrage of the University of Chicago and LINDO Systems (and who took an introductory operations research course from me 50 years ago) provided LINGO and LINDO for the book's website. He also supervised the further development of LINGO/LINDO files for the various chapters as well as providing tutorial material for the book's website. Another long-time friend, Bjarni Kristjansson (who heads Maximal Software), did the same thing for the MPL/Solvers files and MPL tutorial material, as well as arranging to provide a student version of MPL and various elite solvers for the book's website. Still another friend, Daniel Flystra (head of Frontline Systems), has arranged to provide users of this book with a free 140-day license to use a student version of his company's exciting new software package, Analytic Solver Platform. These four individuals and their companies—Accelet Corporation, LINDO Systems, Maximal Software, and Frontline Systems—have made an invaluable contribution to this book.

I also am excited about the partnership with INFORMS that began with the 9th edition. Students can benefit greatly by reading about top-quality applications of operations research. This preeminent professional OR society is enabling this by providing a link to the articles in *Interfaces* that describe the applications of OR that are summarized in the application vignettes and other selected references of award winning OR applications provided in the book.

It was a real pleasure working with McGraw-Hill's thoroughly professional editorial and production staff, including Raghu Srinivasan (Global Publisher), Kathryn Neubauer Carney (the Developmental Editor during most of the development of this edition), Vincent Bradshaw (the Developmental Editor for the completion of this edition), and Mary Jane Lampe (Content Project Manager).

Just as so many individuals made important contributions to this edition, I would like to invite each of you to start contributing to the next edition by using my email address below to send me your comments, suggestions, and errata to help me improve the book in the future. In giving my email address, let me also assure instructors that I will continue to follow the policy of not providing solutions to problems and cases in the book to anybody (including your students) who contacts me.

Enjoy the book.

Frederick S. Hillier
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CHAPTER 1

Introduction

1.1 THE ORIGINS OF OPERATIONS RESEARCH

Since the advent of the industrial revolution, the world has seen a remarkable growth in the size and complexity of organizations. The artisans' small shops of an earlier era have evolved into the billion-dollar corporations of today. An integral part of this revolutionary change has been a tremendous increase in the division of labor and segmentation of management responsibilities in these organizations. The results have been spectacular. However, along with its blessings, this increasing specialization has created new problems, problems that are still occurring in many organizations. One problem is a tendency for the many components of an organization to grow into relatively autonomous empires with their own goals and value systems, thereby losing sight of how their activities and objectives mesh with those of the overall organization. What is best for one component frequently is detrimental to another, so the components may end up working at cross purposes. A related problem is that as the complexity and specialization in an organization increase, it becomes more and more difficult to allocate the available resources to the various activities in a way that is most effective for the organization as a whole. These kinds of problems and the need to find a better way to solve them provided the environment for the emergence of **operations research** (commonly referred to as **OR**).

The roots of OR can be traced back many decades,[1] when early attempts were made to use a scientific approach in the management of organizations. However, the beginning of the activity called *operations research* has generally been attributed to the military services early in World War II. Because of the war effort, there was an urgent need to allocate scarce resources to the various military operations and to the activities within each operation in an effective manner. Therefore, the British and then the U.S. military management called upon a large number of scientists to apply a scientific approach to dealing with this and other strategic and tactical problems. In effect, they were asked to do *research on* (military) *operations.* These teams of scientists were the first OR teams. By developing effective methods of using the new tool of radar, these teams were instrumental in winning the Air Battle of Britain. Through their research on how to better manage convoy and antisubmarine operations, they also played a major role in winning the Battle of the North Atlantic. Similar efforts assisted the Island Campaign in the Pacific.

[1]Selected Reference 7 provides an entertaining history of operations research that traces its roots as far back as 1564 by describing a considerable number of scientific contributions from 1564 to 2004 that influenced the subsequent development of OR. Also see Selected References 1 and 6 for further details about this history.

When the war ended, the success of OR in the war effort spurred interest in applying OR outside the military as well. As the industrial boom following the war was running its course, the problems caused by the increasing complexity and specialization in organizations were again coming to the forefront. It was becoming apparent to a growing number of people, including business consultants who had served on or with the OR teams during the war, that these were basically the same problems that had been faced by the military but in a different context. By the early 1950s, these individuals had introduced the use of OR to a variety of organizations in business, industry, and government. The rapid spread of OR soon followed. (Selected Reference 6 recounts the development of the field of operations research by describing the lives and contributions of 43 OR pioneers.)

At least two other factors that played a key role in the rapid growth of OR during this period can be identified. One was the substantial progress that was made early in improving the techniques of OR. After the war, many of the scientists who had participated on OR teams or who had heard about this work were motivated to pursue research relevant to the field; important advancements in the state of the art resulted. A prime example is the *simplex method* for solving linear programming problems, developed by George Dantzig in 1947. Many of the standard tools of OR, such as linear programming, dynamic programming, queueing theory, and inventory theory, were relatively well developed before the end of the 1950s.

A second factor that gave great impetus to the growth of the field was the onslaught of the *computer revolution.* A large amount of computation is usually required to deal most effectively with the complex problems typically considered by OR. Doing this by hand would often be out of the question. Therefore, the development of electronic digital computers, with their ability to perform arithmetic calculations millions of times faster than a human being can, was a tremendous boon to OR. A further boost came in the 1980s with the development of increasingly powerful personal computers accompanied by good software packages for doing OR. This brought the use of OR within the easy reach of much larger numbers of people, and this progress further accelerated in the 1990s and into the 21st century. For example, the widely used spreadsheet package, Microsoft Excel, provides a Solver that will solve a variety of OR problems.Today, literally millions of individuals have ready access to OR software. Consequently, a whole range of computers from mainframes to laptops now are being routinely used to solve OR problems, including some of enormous size.

1.2 THE NATURE OF OPERATIONS RESEARCH

As its name implies, operations research involves "research on operations." Thus, operations research is applied to problems that concern how to conduct and coordinate the *operations* (i.e., the *activities*) within an organization. The nature of the organization is essentially immaterial, and in fact, OR has been applied extensively in such diverse areas as manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services, to name just a few. Therefore, the breadth of application is unusually wide.

The *research* part of the name means that operations research uses an approach that resembles the way research is conducted in established scientific fields. To a considerable extent, the *scientific method* is used to investigate the problem of concern. (In fact, the term *management science* sometimes is used as a synonym for operations research.) In particular, the process begins by carefully observing and formulating the problem, including gathering all relevant data. The next step is to construct a scientific (typically mathematical) model that attempts to abstract the essence of the real problem. It is then hypothesized that this model is a sufficiently precise representation of the essential features of the situation

that the conclusions (solutions) obtained from the model are also valid for the real problem. Next, suitable experiments are conducted to test this hypothesis, modify it as needed, and eventually verify some form of the hypothesis. (This step is frequently referred to as *model validation.*) Thus, in a certain sense, operations research involves creative scientific research into the fundamental properties of operations. However, there is more to it than this. Specifically, OR is also concerned with the practical management of the organization. Therefore, to be successful, OR must also provide positive, understandable conclusions to the decision maker(s) when they are needed.

Still another characteristic of OR is its broad viewpoint. As implied in the preceding section, OR adopts an organizational point of view. Thus, it attempts to resolve the conflicts of interest among the components of the organization in a way that is best for the organization as a whole. This does not imply that the study of each problem must give explicit consideration to all aspects of the organization; rather, the objectives being sought must be consistent with those of the overall organization.

An additional characteristic is that OR frequently attempts to search for a *best* solution (referred to as an *optimal* solution) for the model that represents the problem under consideration. (We say *a* best instead of *the* best solution because multiple solutions may be tied as best.) Rather than simply improving the status quo, the goal is to identify a best possible course of action. Although it must be interpreted carefully in terms of the practical needs of management, this "search for optimality" is an important theme in OR.

All these characteristics lead quite naturally to still another one. It is evident that no single individual should be expected to be an expert on all the many aspects of OR work or the problems typically considered; this would require a group of individuals having diverse backgrounds and skills. Therefore, when a full-fledged OR study of a new problem is undertaken, it is usually necessary to use a *team approach.* Such an OR team typically needs to include individuals who collectively are highly trained in mathematics, statistics and probability theory, economics, business administration, computer science, engineering and the physical sciences, the behavioral sciences, and the special techniques of OR. The team also needs to have the necessary experience and variety of skills to give appropriate consideration to the many ramifications of the problem throughout the organization.

1.3 THE RISE OF ANALYTICS TOGETHER WITH OPERATIONS RESEARCH

There has been great buzz throughout the business world in recent years about something called **analytics** (or *business analytics*) and the importance of incorporating analytics into managerial decision making. The primary impetus for this buzz was a series of articles and books by Thomas H. Davenport, a renowned thought-leader who has helped hundreds of companies worldwide to revitalize their business practices. He initially introduced the concept of analytics in the January 2006 issue of the *Harvard Business Review* with an article, "Competing on Analytics," that now has been named as one of the ten must-read articles in that magazine's 90-year history. This article soon was followed by two best-selling books entitled *Competing on Analytics: The New Science of Winning* and *Analytics at Work: Smarter Decisions, Better Results*. (See Selected References 2 and 3 at the end of the chapter for the citations.)

So what is analytics? The short (but oversimplified) answer is that it is basically operations research by another name. However, there are some differences in their relative emphases. Furthermore, the strengths of the analytics approach are likely to be increasingly incorporated into the OR approach as time goes on, so it will be instructive to describe analytics a little further.

Analytics fully recognizes that we have entered into the era of *big data* where massive amounts of data now are commonly available to many businesses and organizations to help guide managerial decision making. The current data surge is coming from sophisticated computer tracking of shipments, sales, suppliers, and customers, as well as email, Web traffic, and social networks. As indicated by the following definition, a primary focus of analytics is on how to make the most effective use of all these data.

> *Analytics* is the scientific process of transforming data into insight for making better decisions.

The application of analytics can be divided into three overlapping categories. One of these is *descriptive analytics*, which involves using innovative techniques to locate the relevant data and identify the interesting patterns in order to better describe and understand what is going on now. One important technique for doing this is called *data mining* (as described in Selected Reference 8). Some analytics professionals who specialize in descriptive analytics are called *data scientists*.

A second (and more advanced) category is *predictive analytics*, which involves using the data to predict what will happen in the future. Statistical forecasting methods, such as those described in Chap. 27 (on the book's website), are prominently used here. Simulation (Chap. 20) also can be useful.

The final (and most advanced) category is *prescriptive analytics*, which involves using the data to prescribe what should be done in the future. The powerful optimization techniques of operations research described in many of the chapters of this book generally are what are used here.

Operations research analysts also often deal with all three of these categories, but not very much with the first one, somewhat more with the second one, and then heavily with the last one. Thus, OR can be thought of as focusing mainly on *advanced analytics*—predictive and prescriptive activities—whereas analytics professionals might get more involved than OR analysts with the entire business process, including what precedes the first category (identifying a need) and what follows the last category (implementation). Looking to the future, the two approaches should tend to merge over time. Because the name *analytics* (or *business analytics*) is more meaningful to most people than the term *operations research*, we might find that *analytics* may eventually replace *operations research* as the common name for this integrated discipline.

Although analytics was initially introduced as a key tool for mainly business organizations, it also can be a powerful tool in other contexts. As one example, analytics (together with OR) played a key role in the 2012 presidential campaign in the United States. The Obama campaign management hired a multi-disciplinary team of statisticians, predictive modelers, data-mining experts, mathematicians, software programmers, and OR analysts. It eventually built an entire analytics department five times as large as that of its 2008 campaign. With all this analytics input, the Obama team launched a full-scale and all-front campaign, leveraging massive amounts of data from various sources to directly micro-target potential voters and donors with tailored messages. The election had been expected to be a very close one, but the Obama "ground game" that had been propelled by descriptive and predictive analytics was given much of the credit for the clear-cut Obama win. Based on this experience, both political parties undoubtedly will make extensive use of analytics in the future in major political campaigns.

Another famous application of analytics is described in the book *Moneyball* (cited in Selected Reference 10) and a subsequent 2011 movie with the same name that is based on this book. They tell the true story of how the Oakland Athletics baseball team achieved great success, despite having one of the smallest budgets in the major leagues, by using various kinds of nontraditional data (referred to as *sabermetrics*) to better evaluate the

potential of players available through a trade or the draft. Although these evaluations often flew in the face of conventional baseball wisdom, both descriptive analytics and predictive analytics were being used to identify overlooked players who could greatly help the team. After witnessing the impact of analytics, many major league baseball teams now have hired analytics professionals. Some other kinds of sports teams also are beginning to use analytics. (Selected References 4 and 5 have 17 articles describing the application of analytics in various sports.)

These and numerous other success stories about the power of analytics and OR together should lead to their ever-increasing use in the future. Meanwhile, OR already has had a powerful impact, as described further in the next section.

1.4 THE IMPACT OF OPERATIONS RESEARCH

Operations research has had an impressive impact on improving the efficiency of numerous organizations around the world. In the process, OR has made a significant contribution to increasing the productivity of the economies of various countries. There now are a few dozen member countries in the International Federation of Operational Research Societies (IFORS), with each country having a national OR society. Both Europe and Asia have federations of OR societies to coordinate holding international conferences and publishing international journals in those continents. In addition, the Institute for Operations Research and the Management Sciences (INFORMS) is an international OR society that is headquartered in the United States. Just as in many other developed countries, OR is an important profession in the United States. According to projections from the U.S. Bureau of Labor Statistics for the year 2013, there are approximately 65,000 individuals working as operations research analysts in the United States with an average salary of about $79,000.

Because of the rapid rise of *analytics* described in the preceding section, INFORMS has embraced analytics as an approach to decision making that largely overlaps and further enriches the OR approach. Therefore, this leading OR society now includes an annual Conference on Business Analytics and Operations Research among its major conferences. It also provides a Certified Analytics Professional credential for those individuals who satisfy certain criteria and pass an examination. In addition, INFORMS publishes many of the leading journals in the field, including one called *Analytics*, and another, called *Interfaces*, regularly publishes articles describing major OR studies and the impact they had on their organizations.

To give you a better notion of the wide applicability of OR, we list some actual applications in Table 1.1 that have been described in *Interfaces*. Note the diversity of organizations and applications in the first two columns. The third column identifies the section where an "application vignette" devotes several paragraphs to describing the application and also references an article that provides full details. (You can see the first of these application vignettes in this section.) The last column indicates that these applications typically resulted in annual savings in the many millions of dollars. Furthermore, additional benefits not recorded in the table (e.g., improved service to customers and better managerial control) sometimes were considered to be even more important than these financial benefits. (You will have an opportunity to investigate these less tangible benefits further in Probs. 1.3-1, 1.3-2, and 1.3-3.) A link to the articles that describe these applications in detail is included on our website, www.mhhe.com/hillier.

Although most routine OR studies provide considerably more modest benefits than the applications summarized in Table 1.1, the figures in the rightmost column of this table do accurately reflect the dramatic impact that large, well-designed OR studies occasionally can have.

An Application Vignette

FedEx Corporation is the world's largest courier delivery services company. Every working day, it delivers many millions of documents, packages, and other items throughout the United States and hundreds of countries and territories around the world. In some cases, these shipments can be guaranteed overnight delivery by 10:30 A.M. the next morning.

The logistical challenges involved in providing this service are staggering. These millions of daily shipments must be individually sorted and routed to the correct general location (usually by aircraft) and then delivered to the exact destination (usually by motorized vehicle) in an amazingly short period of time. How is all this possible?

Operations research (OR) is the technological engine that drives this company. Ever since its founding in 1973, OR has helped make its major business decisions, including equipment investment, route structure, scheduling, finances, and location of facilities. After OR was credited with literally saving the company during its early years, it became the custom to have OR represented at the weekly senior management meetings and, indeed, several of the senior corporate vice presidents have come up from the outstanding FedEx OR group.

FedEx has come to be acknowledged as a world-class company. It routinely ranks among the top companies on *Fortune Magazine*'s annual listing of the "World's Most Admired Companies and this same magazine named the firm as one of the top 100 companies to work for in 2013." It also was the first winner (in 1991) of the prestigious prize now known as the INFORMS Prize, which is awarded annually for the effective and repeated integration of OR into organizational decision making in pioneering, varied, novel, and lasting ways. The company's great dependence on OR has continued to the present day.

Source: R. O. Mason, J. L. McKenney, W. Carlson, and D. Copeland, "Absolutely, Positively Operations Research: The Federal Express Story," *Interfaces,* **27**(2): 17–36, March—April 1997. (A link to this article is provided on our website, www.mhhe.com/hillier.)

TABLE 1.1 Applications of operations research to be described in application vignettes

Organization	Area of Application	Section	Annual Savings
Federal Express	Logistical planning of shipments	1.4	Not estimated
Continental Airlines	Reassign crews to flights when schedule disruptions occur	2.2	$40 million
Swift & Company	Improve sales and manufacturing performance	3.1	$12 million
Memorial Sloan-Kettering Cancer Center	Design of radiation therapy	3.4	$459 million
Welch's	Optimize use and movement of raw materials	3.5	$150,000
INDEVAL	Settle all securities transactions in Mexico	3.6	$150 million
Samsung Electronics	Reduce manufacturing times and inventory levels	4.3	$200 million more revenue
Pacific Lumber Company	Long-term forest ecosystem management	7.2	$398 million NPV
Procter & Gamble	Redesign the production and distribution system	9.1	$200 million
Canadian Pacific Railway	Plan routing of rail freight	10.3	$100 million
Hewlett-Packard	Product portfolio management	10.5	$180 million
Norwegian companies	Maximize flow of natural gas through offshore pipeline network	10.5	$140 million
United Airlines	Reassign airplanes to flights when disruptions occur	10.6	Not estimated
U.S. Military	Logistical planning of Operations Desert Storm	11.3	Not estimated
MISO	Administer the transmission of electricity in 13 states	12.2	$700 million
Netherlands Railways	Optimize operation of a railway network	12.2	$105 million
Taco Bell	Plan employee work schedules at restaurants	12.5	$13 million
Waste Management	Develop a route-management system for trash collection and disposal	12.7	$100 million
Bank Hapoalim Group	Develop a decision-support system for investment advisors	13.1	$31 million more revenue
DHL	Optimize the use of marketing resources	13.10	$22 million
Sears	Vehicle routing and scheduling for home services and deliveries	14.2	$42 million

TABLE 1.1 Applications of operations research to be described in application vignettes *(contd)*

Organization	Area of Application	Section	Annual Savings
Intel Corporation	Design and schedule the product line	14.4	Not estimated
Conoco-Phillips	Evaluate petroleum exploration projects	16.2	Not estimated
Workers' Compensation Board	Manage high-risk disability claims and rehabilitation	16.3	$4 million
Westinghouse	Evaluate research-and-development projects	16.4	Not estimated
KeyCorp	Improve efficiency of bank teller service	17.6	$20 million
General Motors	Improve efficiency of production lines	17.9	$90 million
Deere & Company	Management of inventories throughout a supply chain	18.5	$1 billion less inventory
Time Inc.	Management of distribution channels for magazines	18.7	$3.5 million more profit
InterContinental Hotels	Revenue management	18.8	$400 million more revenue
Bank One Corporation	Management of credit lines and interest rates for credit cards	19.2	$75 million more profit
Merrill Lynch	Pricing analysis for providing financial services	20.2	$50 million more revenue
Sasol	Improve the efficiency of its production processes	20.5	$23 million
FAA	Manage air traffic flows in severe weather	20.5	$200 million

1.5 ALGORITHMS AND OR COURSEWARE

An important part of this book is the presentation of the major **algorithms** (systematic solution procedures) of OR for solving certain types of problems. Some of these algorithms are amazingly efficient and are routinely used on problems involving hundreds or thousands of variables. You will be introduced to how these algorithms work and what makes them so efficient. You then will use these algorithms to solve a variety of problems on a computer. The **OR Courseware** contained on the book's website (www.mhhe.com/hillier) will be a key tool for doing all this.

One special feature in your OR Courseware is a program called **OR Tutor.** This program is intended to be your personal tutor to help you learn the algorithms. It consists of many *demonstration examples* that display and explain the algorithms in action. These "demos" supplement the examples in the book.

In addition, your OR Courseware includes a special software package called **Interactive Operations Research Tutorial,** or **IOR Tutorial** for short. Implemented in Java, this innovative package is designed specifically to enhance the learning experience of students using this book. IOR Tutorial includes many *interactive procedures* for executing the algorithms interactively in a convenient format. The computer does all the routine calculations while you focus on learning and executing the logic of the algorithm. You should find these interactive procedures a very efficient and enlightening way of doing many of your homework problems. IOR Tutorial also includes a number of other helpful procedures, including some *automatic procedures* for executing algorithms automatically and several procedures that provide graphical displays of how the solution provided by an algorithm varies with the data of the problem.

In practice, the algorithms normally are executed by commercial software packages. We feel that it is important to acquaint students with the nature of these packages that they will be using after graduation. Therefore, your OR Courseware includes a wealth of material to introduce you to four particularly popular software packages described next. Together, these packages will enable you to solve nearly all the OR models encountered in this book very efficiently. We have added our own *automatic procedures* to IOR Tutorial in a few cases where these packages are not applicable.

A very popular approach now is to use today's premier spreadsheet package, Microsoft Excel, to formulate small OR models in a spreadsheet format. Included with standard Excel is an add-in, called **Solver** (a product of Frontline Systems, Inc.), that can be used to solve many of these models. Your OR Courseware includes separate Excel files for nearly every chapter in this book. Each time a chapter presents an example that can be solved using Excel, the complete spreadsheet formulation and solution is given in that chapter's Excel files. For many of the models in the book, an *Excel template* also is provided that already includes all the equations necessary to solve the model.

New with this edition of the textbook is a powerful software package from Frontline Systems called **Analytic Solver Platform for Education (ASPE)**, which is fully compatible with Excel and Excel's Solver. The recently released Analytic Solver Platform combines all the capabilities of three other popular products from Frontline Systems: (1) Premium Solver Platform (a powerful spreadsheet optimizer that includes five solvers for linear, mixed-integer, nonlinear, non-smooth, and global optimization), (2) Risk Solver Pro (for simulation and risk analysis), and (3) XLMiner (an Excel-based tool for data mining and forecasting). It also has the ability to solve optimization models involving uncertainty and recourse decisions, perform sensitivity analysis, and construct decision trees. It even has an ultra-high-performance linear mixed-integer optimizer. The student version of Analytic Solver Platform retains all these capabilities when dealing with smaller problems. Among the special features of ASPE that are highlighted in this book are a greatly enhanced version of the basic Solver included with Excel (as described in Sec. 3.5), the ability to build decision trees within Excel (as described in Sec. 16.5), and tools to build simulation models within Excel (as described in Sec. 20.6).

After many years, **LINDO** (and its companion modeling language **LINGO**) continues to be a popular OR software package. Student versions of LINDO and LINGO now can be downloaded free from the Web at www.lindo.com. This student version also is provided in your OR Courseware. As for Excel, each time an example can be solved with this package, all the details are given in a LINGO/LINDO file for that chapter in your OR Courseware.

When dealing with large and challenging OR problems, it is common to also use a *modeling system* to efficiently formulate the mathematical model and enter it into the computer. **MPL** is a user-friendly modeling system that includes a considerable number of elite solvers for solving such problems very efficiently. These solvers include CPLEX, GUROBI, CoinMP, and SULUM for linear and integer programming (Chaps. 3-10 and 12), as well as CONOPT for convex programming (part of Chap. 13) and LGO for global optimization (Sec. 13.10), among others. A student version of MPL, along with the student version of its solvers, is available free by downloading it from the Web. For your convenience, we also have included this student version (including the six solvers just mentioned) in your OR Courseware. Once again, all the examples that can be solved with this package are detailed in MPL/Solvers files for the corresponding chapters in your OR Courseware. Furthermore, academic users can apply to receive full-sized versions of MPL, CPLEX, and GUROBI by going to their respective websites.[2] This means that any academic users (professors or students) now can obtain professional versions of MPL with CPLEX and GUROBI for use in their coursework.

We will further describe these four software packages and how to use them later (especially near the end of Chaps. 3 and 4). Appendix 1 also provides documentation for the OR Courseware, including OR Tutor and IOR Tutorial.

To alert you to relevant material in OR Courseware, the end of each chapter from Chap. 3 onward has a list entitled *Learning Aids for This Chapter on our Website.* As

[2]MPL: http://www.maximalsoftware.com/academic; CPLEX: http://www-03.ibm.com/ibm/university/academic/pub/page/ban_ilog_programming; GUROBI: http://www.gurobi.com/products/licensing-and-pricing/academic-licensing

explained at the beginning of the problem section for each of these chapters, symbols also are placed to the left of each problem number or part where any of this material (including demonstration examples and interactive procedures) can be helpful.

Another learning aid provided on our website is a set of **Solved Examples** for each chapter (from Chap. 3 onward). These complete examples supplement the examples in the book for your use as needed, but without interrupting the flow of the material on those many occasions when you don't need to see an additional example. You also might find these supplementary examples helpful when preparing for an examination. We always will mention whenever a supplementary example on the current topic is included in the Solved Examples section of the book's website. To make sure you don't overlook this mention, we will boldface the words **additional example** (or something similar) each time.

The website also includes a glossary for each chapter.

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PROBLEMS

1.3-1. Select one of the applications of operations research listed in Table 1.1. Read the article that is referenced in the application vignette presented in the section shown in the third column. (A link to all these articles is provided on our website, www.mhhe.com/hillier.) Write a two-page summary of the application and the benefits (including nonfinancial benefits) it provided.

1.3-2. Select three of the applications of operations research listed in Table 1.1. For each one, read the article that is referenced in the application vignette presented in the section shown in the third column. (A link to all these articles is provided on our website, www.mhhe.com/hillier.) For each one, write a one-page summary of the application and the benefits (including nonfinancial benefits) it provided.

1.3-3. Read the referenced article that fully describes the OR study summarized in the application vignette presented in Sec. 1.4. List the various financial and nonfinancial benefits that resulted from this study.